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Nonsimilarity solutions for mixed convection from horizontal surfaces in a porous medium—variable wall temperature

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Abstract—Mixed convection in a porous medium from horizontal surfaces with variable wall temperature distribution is analyzed. The entire mixed convection regime is divided into two regions. The first region covers the forced convection dominated regime where the dimensionless parameter $\xi_f = Ra_x/Pe_x^{3/2}$ is found to characterize the effect of buoyancy forces on the forced convection. The second region covers the free convection dominated regime where the dimensionless parameter $\xi_n = Pe_x/Ra_x^{2/3}$ is found to characterize the effect of the forced flow on the free convection. To obtain the solution that covers the entire mixed convection regime, the solution of the first region is carried out for $\xi_f = 0$, the pure forced convection limit, to $\xi_f = 1$ and the solution of the second region is carried out for $\xi_n = 0$, the pure free convection limit, to $\xi_n = 1$. The two solutions meet and match at $\xi_f = \xi_n = 1$. Numerical results for different wall temperature variations are presented. In addition, correlation equations for the local and average Nusselt numbers are obtained.

INTRODUCTION

IN GENERAL the flow and thermal field in mixed convection from surfaces in a porous medium are nonsimilar, with the exception of certain cases with particular boundary conditions and free stream velocity distribution [1, 2]. For example, the case of variable wall temperature does not provide a similar solution and solutions must be sought by a nonsimilar method. For such a problem the approximation method of local similarity can be utilized. However, the local similarity solution suffers from inaccuracy with an error of 10–15% [3, 4]. To obtain more accurate results for nonsimilar boundary layer systems the local nonsimilarity solution method can be used [5]. This method has been applied recently [3, 6] for obtaining solutions for nonsimilar natural and mixed convection problems in porous media. Mixed convection from a horizontal plate with prescribed wall temperature in a porous medium is found to be characterized by the dimensionless parameter $\xi_f = Ra_x/Pe_x^{3/2}$ [1, 7]. This parameter covers the forced convection dominated regime; however, it is not possible to cover the entire mixed convection regime because of the singularity it suffers at the limiting end of pure free convection. Nakayama and Pop [8] proposed a unified similarity transformation to the governing equations to overcome the problem of singularity at the other limiting end of the regime. Although the local nonsimilarity method can provide accurate results, it is an approximate method because some of the higher order terms in the governing equations are neglected. A more exact solution for the nonsimilar boundary layer systems can be obtained by using a finite difference method [9, 10]. In this method one needs to carry out the solution from $\xi_f = 0$ to the downstream location at a particular value of ξ_f. Thus, to cover an appreciable region of the forced convection dominated regime a large computer time is needed. To overcome such a problem in the present work, the entire mixed convection regime is divided into two regions as was done by Aldoss *et al.* [4]. The first region covers the forced convection dominated regime where $\xi_f = Ra_x/Pe_x^{3/2}$ is used to characterize the effect of the buoyancy force on the forced convection. The second region covers the free convection dominated regime with $\xi_n = Pe_x/Ra_x^{2/3}$ as the dimensionless parameter that characterizes the effect of forced flow on the free convection. The first system is solved to cover the region between $\xi_f = 0$ and 1 and the second system is solved to cover the region between $\xi_n = 0$ and 1. The two solutions meet and match at $\xi_f = \xi_n = 1$ [4]. Each of these solutions provides a half of the total solution for the entire

NOMENCLATURE

C_{fx}	local friction factor
f	dimensionless stream function
$h(x)$	local heat transfer coefficient
$\bar{h}$	average heat transfer coefficient, $(1/L)\int_0^L h(x)\,dx$
k	thermal conductivity
K	permeability coefficient of the porous medium
L	length of the plate
Nu_x	local Nusselt number, hx/k
$\overline{Nu}$	average Nusselt number, $\bar{h}L/k$
Pe_x	local Peclet number, $u_\infty x/\alpha$
q_w	local surface heat flux
Ra_x	local Rayleigh number, $g\beta[T_w(x)-T_\infty]Kx/(\nu\alpha)$
Re	Reynolds number, $u_\infty x/\nu$
T	temperature
T_∞	free stream temperature
T_w	wall temperature
u, v	velocity components in x- and y-direction
u_∞	free stream velocity
x, y	axial and normal coordinates.

Greek symbols

α	effective thermal diffusivity of saturated porous medium
β	volumetric coefficient of thermal expansion
δ	boundary layer thickness
η	pseudo-similarity variable
θ	dimensionless temperature
μ	dynamic viscosity
ν	kinematic viscosity
ξ_f	nonsimilarity parameter for the forced convection dominated regime
ξ_n	nonsimilarity parameter for the free convection dominated regime
ρ	fluid density
ψ	stream function.

Subscripts

f	forced convection dominated condition
n	free convection dominated condition
x, y, ξ_f, ξ_n	partial derivatives with respect to x, y, ξ_f, and ξ_n, respectively.

mixed convection regime. Numerical results for different wall temperature distribution are presented.

ANALYSIS

Consider the mixed convection from an impermeable horizontal plate at the bottom of a porous medium where the plate is heated and has variable wall temperature. The Darcy model is used, which is valid under the conditions of low velocities and small pores of porous matrix [11]. Also, the assumption of slip velocity at the wall is imposed, which has a smaller effect on the heat transfer results as the distance from the leading edge increases [12]. The axial and normal coordinates are x and y, and the corresponding Darcian velocity components are u and v, respectively. The gravitational acceleration g is acting downward in the direction opposite to the y coordinate. The properties of the fluid and the porous medium are assumed to be constant and isotropic. By invoking the Boussinesq and the boundary layer approximations, the governing equations are given by [13]

$$u_x + v_y = 0 \tag{1}$$

$$\psi_{yy} = -(K\rho_\infty g\beta/\mu)T_x \tag{2}$$

$$T_{yy} = (1/\alpha)(\psi_y T_x - \psi_x T_y). \tag{3}$$

In the above equations, the stream function ψ satisfies the continuity equation with $u = \psi_y$ and $v = -\psi_x$; T is the temperature; ρ, μ, and β are the density, viscosity, and the thermal expansion coefficient of the convecting fluid, respectively; K is the permeability of the porous medium; and α is the equivalent thermal diffusivity of the porous medium. With power-law variation in the wall temperature, the boundary conditions can be written as

$$y = 0: T_w - T_\infty = ax^n, v = 0$$
$$y = \infty: T = T_\infty, u = u_\infty \tag{4}$$

where a and n are prescribed constants. Note that $n = 0$ correspond to the case of uniform wall temperature.

A. *Forced convection dominated regime*

In this regime the following dimensionless variables are introduced:

$$\eta = (y/x)Pe_x^{1/2}, \quad \xi_f = \xi_f(x) \tag{5}$$

$$\psi = \alpha Pe_x^{1/2} f(\xi_f, \eta),$$
$$\theta(\xi_f, \eta) = (T - T_\infty)/[T_w(x) - T_\infty]. \tag{6}$$

The governing equations and boundary conditions, equations (1)–(4), can then be transformed into

$$f'' + \xi_f[n\theta - (\eta/2)\theta'] = -(n-1/2)\xi_f^2\theta_{\xi_f} \tag{7}$$

$$\theta'' + (1/2)f\theta' - nf'\theta = (n-1/2)\xi_f(f'\theta_{\xi_f} - \theta' f_{\xi_f}) \tag{8}$$

$$f(\xi_f, 0) + 2(n-1/2)\xi_f f_{\xi_f}(\xi_f, 0) = 0 \quad \text{or} \quad f(\xi_f, 0) = 0,$$

$$\theta(\xi_f, 0) = 1, \quad f'(\xi_f, \infty) = 1, \quad \theta(\xi_f, \infty) = 0 \tag{9}$$

where

$$\xi_f(x) = Ra_x/Pe_x^{3/2} \tag{10}$$

and the primes denote partial differentiations with respect to η.

In the above system of equations, the dimensionless parameter ξ_f is a measure of the buoyancy effect on forced convection. The case of $\xi_f = 0$ corresponds to pure forced convection and the limiting case of $\xi_f = \infty$ corresponds to pure free convection. The above system of equations (7)–(9) is solved for the region covered by $\xi_f = 0$–1 to provide the first half of the total solution of the mixed convection regime.

Some of the physical quantities of interest include the velocity components u and v in the x and y directions, the local friction factor C_{fx} (defined as $\tau_w/(\rho u_\infty^2)/2$, where $\tau_w = \mu(u_y)_{y=0}$), and the local Nusselt number $Nu_x = hx/k$, where $h = q_w/[T_w(x) - T_\infty]$. They are given by

$$u = u_\infty f'(\xi_f, \eta) \tag{11}$$

$$v = -(\alpha/x)Pe_x^{1/2}[(1/2)f(\xi_f,\eta) - (\eta/2)f'(\xi_f,\eta) + (n-1/2)\xi_f f_{\xi_f}] \tag{12}$$

$$C_{fx}\, Pr^{-1}\, Pe_x^{1/2} = 2f''(\xi_f, 0) \tag{13}$$

and

$$Nu_x\, Pe_x^{-1/2} = -\theta'(\xi_f, 0). \tag{14}$$

The average Nusselt number $\overline{Nu}$ can be evaluated by finding the average heat transfer coefficient $\bar{h}$ from the local Nusselt number expression, equation (14). The end result is

$$\overline{Nu}\, Pe_L^{-1/2} = \frac{-2}{2n-1}\xi_{f_L}^{-1/(2n-1)}\int_0^{\xi_{f_L}} \theta'(\xi_f, 0)\xi_f^{(2-2n)/(2n-1)}\, d\xi_f \tag{15}$$

where Pe_L and ξ_{f_L} are Pe_x and ξ_f evaluated at $\xi = L$.

B. *Free convection dominated regime*

For buoyancy dominated regime the following dimensionless variables are introduced in the transformation

$$\eta = (y/x)Ra_x^{1/3}, \quad \xi_n = \xi_n(x) \tag{16}$$

$$\psi = \alpha\, Ra_x^{1/3} f(\xi_n, \eta),$$

$$\theta(\xi_n, \eta) = (T - T_\infty)/[T_w(x) - T_\infty]. \tag{17}$$

Substituting equations (16) and (17) into the governing equations (1)–(4) leads to

$$f'' + n\theta + [(n-2)/3]\eta\theta' = -[(1-2n)/3]\xi_n\theta_{\xi_n} \tag{18}$$

$$\theta'' + [(n+1)/3]f\theta' - nf'\theta = [(1-2n)/3]\xi_n(f'\theta_{\xi_n} - \theta' f_{\xi_n}) \tag{19}$$

$$(n+1)f(\xi_n, 0) + (1-2n)\xi_n f_{\xi_n}(\xi_n, 0) = 0 \text{ or } f(\xi_n, 0) = 0$$

$$\theta(\xi_n, 0) = 1, \quad f'(\xi_n, \infty) = \xi_n, \quad \theta(\xi_n, \infty) = 0 \tag{20}$$

where

$$\xi_n(x) = Pe_x/Ra_x^{2/3} \tag{21}$$

and the primes in equations (18)–(20) denote the partial differentiations with respect to η.

Note that the ξ_n parameter here is a measure of the forced flow effect on free convection. The case of $\xi_n = 0$ corresponds to pure free convection and the limiting case of $\xi_n = \infty$ corresponds to pure forced convection. The above system of equations (18)–(20) is solved over the region covered by $\xi_n = 0$–1 to provide the other half of the solution for the entire mixed convection regime.

The velocity components u and v, the local friction factor, and the local Nusselt number have the expressions

$$u = (\alpha/x)Ra_x^{2/3} f'(\xi_n, \eta) \tag{22}$$

$$v = -(\alpha/x)Ra_x^{1/3}\{[(n+1)/3]f(\xi_n,\eta) + [(n-2)/3]\eta f'(\xi_n,\eta) + [(1-2n)/3]\xi_n f_{\xi_n}\} \tag{23}$$

$$C_f\, Pe_x^2\, Pr^{-1}\, Ra_x^{-1} = 2f''(\xi_n, 0) \tag{24}$$

and

$$Nu_x\, Ra_x^{-1/3} = -\theta'(\xi_n, 0). \tag{25}$$

The corresponding average Nusselt number $\overline{Nu}$ can be evaluated as in the previous case and has the expression

$$\overline{Nu}\, Ra_L^{-1/3} = \frac{-3}{1-2n}\xi_{n_L}^{-(n+1)/(1-2n)}\int_0^{\xi_{n_L}} \theta'(\xi_n, 0)\xi_n^{3n/(1-2n)}\, d\xi_n \tag{26}$$

where Ra_L and ξ_{n_L} and Ra_x and ξ_n at $x = L$.

The two systems of partial differential equations, equations (7)–(9) and (18)–(20), have the same general form. Thus, they can be solved using the same method. The finite-difference method of Keller as described by Cebeci and Bradshaw [9] is used and the details of such solution method are not presented here to conserve space. The complete solution for the entire mixed convection regime is constructed from the two separate solutions of the above two systems of equations.

RESULTS AND DISCUSSION

Results are presented for the values of the exponent $0.5 \leqslant n \leqslant 2$. These exponent values are found to provide physically realistic problems for the case of variable wall temperature of the power-law form. The criteria in determining the range of n values are based on the requirements that the streamwise velocity and the boundary layer thickness must increase or remain constant with increasing distance from the leading edge as long as the wall temperature at $x > 0$ is higher than that of the surroundings [13]. From equation (16) the boundary layer thickness, which is of order of y, varies like $x^{-(n-2)/3}$ and from equation (22) u varies like $x^{(2n-1)/3}$. Thus, the condition on the exponent n is $0.5 \leqslant n \leqslant 2.0$.

Results for $\theta(\xi_f, \eta)$ and $f'(\xi_f, \eta)$, the temperature and the velocity profiles, are presented in Figs. 1 and 2 for different values of ξ_f and n. Figure 1 shows that as n increases the thermal boundary layer thickness decreases and that for a particular value of ξ_f the temperature gradient at the wall increases as n increases, resulting in a higher heat transfer rate at a higher value of n. Also, as the buoyancy parameter ξ_f increases the temperature gradient at the wall increases. This means that higher heat transfer rate is expected at a higher value of ξ_f. The effect of increasing n value on the velocity profiles is evident from Fig. 2, where it can be seen that as n increases the momentum boundary layer thickness decreases. In addition, as expected, an increase in ξ_f increases the slip velocity at the wall.

Values of $-\theta'(\xi_f, 0)$ from solutions of the forced convection dominated system and $-\theta'(\xi_n, 0)$, from solutions of the free convection dominated system at selected values of ξ_f and ξ_n are listed in Table 1 for different values of n. The local Nusselt number distribution for different values of n are shown in Fig. 3 for the entire mixed convection regime in terms of $Nu_x\, Pe_x^{-1/2}$. It is clear from Fig. 3 that the two solutions from the forced convection end and from the free convection end meet and match nicely over the mixed convection regime. It is seen from Fig. 3 that higher Nusselt number occurs at higher values of n and ξ_f, which means that increasing buoyancy force gives rise to an increase in the rate of heat transfer. The local Nusselt number expression from the free convection end, equation (25), can be written in the form

$$Nu_x\, Pe_x^{-1/2} = [-\theta'(\xi_n, 0)]\xi_f^{1/3} \qquad (27)$$

where $\xi_f = \xi_n^{-3/2}$ or $\xi_n = \xi_f^{-2/3}$. The domain for pure forced convection, mixed convection, and pure free convection can be established from the present results based on a 5% departure in the local Nusselt number from pure forced convection limit or from pure free convection limit. These values are listed in Table 2.

For practical purposes, correlation equations were developed for the local Nusselt numbers. By using the least square fitting technique the local Nusselt number for pure forced convection in the range of $0.5 \leqslant n \leqslant 2$ can be correlated by

$$Nu_f = f_1(n)\, Pe_x^{1/2} \qquad (28)$$

where

$$f_1(n) = 0.582 + 0.685n - 0.167n^2 + 0.0285n^3. \qquad (29)$$

For the case of pure free convection, the corres-

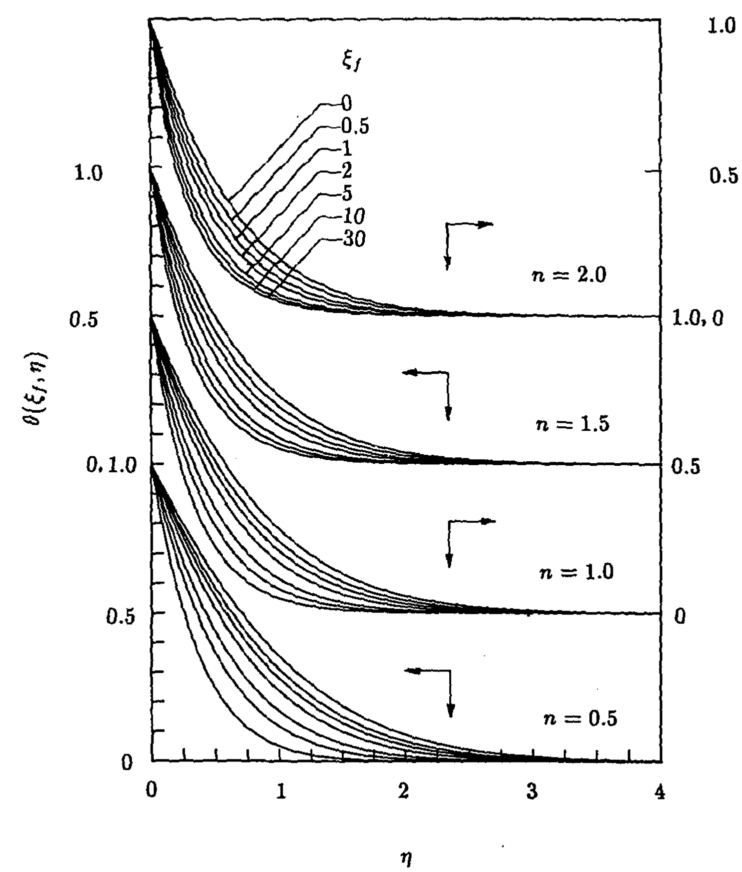

FIG. 1. Dimensionless temperature profiles at selected values of ξ_f and n.

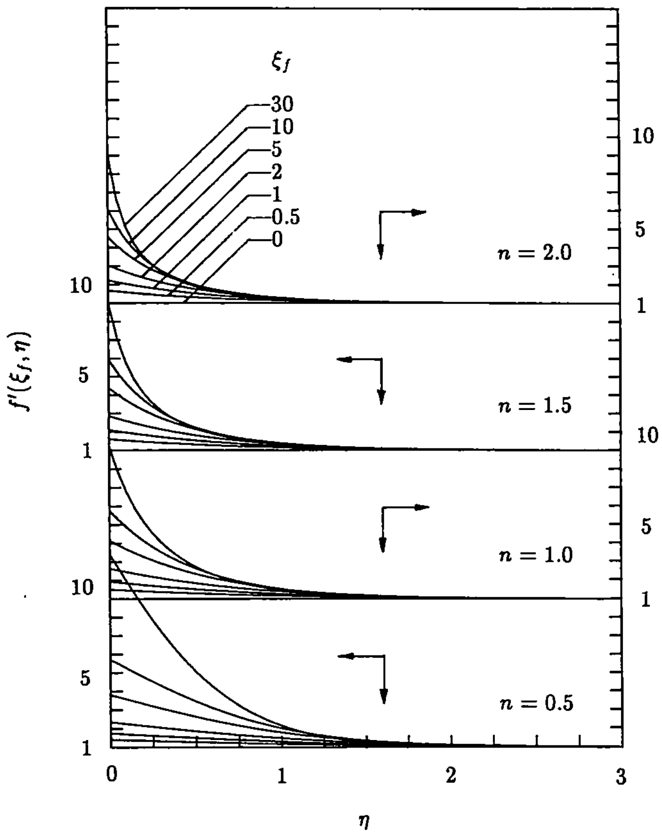

FIG. 2. Velocity profiles at selected values of ξ_f and n.

Table 1. Values of $-\theta'(\xi_f, 0)$, $-\theta'(\xi_n, 0)$ at selected values of ξ_f and ξ_n for different values of n

	$-\theta'(\xi_f, 0)$			
ξ_f	$n = 0.5$	$n = 1.0$	$n = 1.5$	$n = 2$
0.0	0.886230E+00	0.112826E+01	0.133299E+01	0.150458E+01
0.1	0.913309E+00	0.117368E+01	0.139162E+01	0.158304E+01
0.2	0.938787E+00	0.121518E+01	0.144782E+01	0.165322E+01
0.3	0.962980E+00	0.125395E+01	0.149987E+01	0.171792E+01
0.4	0.985962E+00	0.129016E+01	0.154808E+01	0.177754E+01
0.5	0.100781E+01	0.132400E+01	0.159273E+01	0.183245E+01
0.6	0.102859E+01	0.135566E+01	0.163412E+01	0.188306E+01
0.7	0.104839E+01	0.138533E+01	0.167254E+01	0.192975E+01
0.8	0.106727E+01	0.141317E+01	0.170828E+01	0.197293E+01
0.9	0.108532E+01	0.143938E+01	0.174165E+01	0.201298E+01
1.0	0.110204E+01	0.146264E+01	0.177029E+01	0.204617E+01
	$-\theta'(\xi_n, 0)$			
ξ_n	$n = 0.5$	$n = 1.0$	$n = 1.5$	$n = 2$
1.0	0.110203E+01	0.146262E+01	0.177016E+01	0.204601E+01
0.9	0.107088E+01	0.142489E+01	0.172747E+01	0.200156E+01
0.8	0.104101E+01	0.138820E+01	0.168541E+01	0.195481E+01
0.7	0.101127E+01	0.135143E+01	0.164311E+01	0.190769E+01
0.6	0.981760E+00	0.131467E+01	0.160065E+01	0.186022E+01
0.5	0.952586E+00	0.127799E+01	0.155806E+01	0.181243E+01
0.4	0.923849E+00	0.124146E+01	0.151541E+01	0.176436E+01
0.3	0.895652E+00	0.120517E+01	0.147275E+01	0.171603E+01
0.2	0.868097E+00	0.116919E+01	0.143013E+01	0.166746E+01
0.1	0.841288E+00	0.113359E+01	0.138762E+01	0.161870E+01
0.0	0.816426E+00	0.109947E+01	0.134586E+01	0.157116E+01

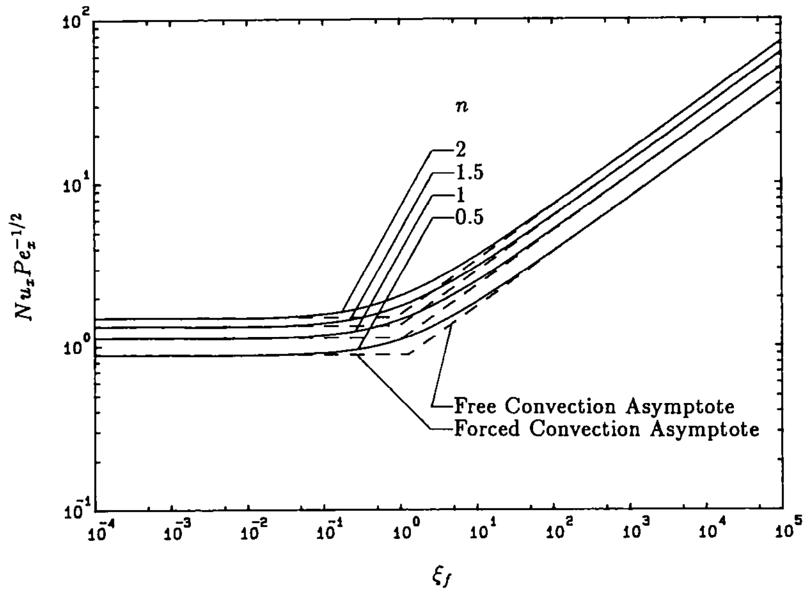

FIG. 3. Local Nusselt number variation for mixed convection with variable wall temperature ($T_w - T_\infty = ax^n$).

ponding correlation equation for the local Nusselt number is given by

$$Nu_n = f_2(n)\, Ra_x^{1/3} \quad (30)$$

where

$$f_2(n) = 0.474+0.762n-0.168n^2+0.0319n^3. \quad (31)$$

Equations (28) and (30) fit the computed results for pure forced convection and pure free convection within an error of less than 5%, respectively.

Following Churchill [14], the correlation equation for the local Nusselt number in mixed convection can be expressed as

$$(Nu_x/Nu_f)^m = 1+(Nu_n/Nu_f)^m. \quad (32)$$

For the present problem the correlation equation for the local mixed convection Nusselt number can be presented by

$$Nu_x Pe_x^{-1/2}/f_1(n) = [1+\{f_2(n)(Ra_x/Pe_x^{3/2})^{1/3}/f_1(n)\}^m]^{1/m}. \quad (33)$$

The corresponding correlation equation for the average mixed convection Nusselt number $\overline{Nu} = \bar{h}L/k$, where $\bar{h}$ is the average heat transfer coefficient over the plate length L, can be presented by

Table 2. Domains of pure forced convection, mixed convection, and pure free convection

Exponent n	Range of $\xi_f = Ra_x/Pe_x^{3/2}$ values for: Forced convection	Mixed convection	Free convection
0.5	0–0.18	0.18–11	11–∞
1.0	0–0.11	0.11–17	17–∞
1.5	0–0.07	0.07–24	24–∞
2.0	0–0.06	0.06–27	27–∞

$$\overline{Nu}\, Pe_L^{-1/2}/2f_1(n) = [1+\{[3/(n+1)]f_2(n)(Ra_L/Pe_L^{3/2})^{1/3}/2f_1(n)\}^m]^{1/m} \quad (34)$$

where Pe_L and Ra_L are Pe_x and Ra_x evaluated at $x = L$. Equation (34) is obtained from equation (33) by knowing the average Nusselt number expressions for pure forced convection $\overline{Nu}_f$ and pure free convection $\overline{Nu}_n$. They are found as

$$\overline{Nu}_f = 2f_1(n)\, Pe_L^{1/2} \quad (35)$$

and

$$\overline{Nu}_n = [3/(n+1)]f_2(n)\, Ra_L^{1/3}. \quad (36)$$

An exponent value of $m = 3$ in equations (33) and (34) is found to correlate the predicted results very well. The maximum deviation between the correlated and the predicted mixed convection Nusselt numbers is found to be less than 5% for the range of $0.5 \leq n \leq 2$ over the entire regime of mixed convection.

CONCLUDING REMARKS

In studying mixed convection from a horizontal plate in a saturated porous medium with variable wall temperature, the analysis is carried out by dividing the entire regime into two regions. The first is forced convection dominated region where the dimensionless parameter $\xi_f = Ra_x/Pe_x^{3/2}$ characterizes the effect of the buoyancy force on the forced convection and the second is the free convection dominated region where the dimensionless parameter $\xi_n = Pe_x/Ra_x^{2/3}$ characterizes the effect of the forced flow on the free convection. Numerical solutions using a finite difference scheme are obtained for the two regions for the range of ξ_f and ξ_n from 0 to 1. The two solutions meet and match at $\xi_f = \xi_n = 1$. By this approach the entire mixed convection regime is covered completely by the two solutions. Numerical results from both regions of solutions are presented for different values of the wall temperature variation. Correlation equations for the local and the average Nusselt numbers are also given.

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